Algebra 2 – Unit 1 Review – Systems of Equations and Inequalities Name KEY

1. Solve each system of equations using any method. Explain which method you used and why.

$y = 4x + 4$

$-8x + 2y = 8$

$-8x + 2(4x+4) = 8$

$\cancel{-8x + 8x} + 8 = 8$

$8 = 8$ ← IDENTITY

INFINITELY MANY SOLUTIONS

LINES COINCIDE (SAME LINE)

(2) $2x - 3y = -5$ > $4x - \cancel{6y} = -10$

(3) $5x + 2y = 16$ > $15x + \cancel{6y} = 48$

$19x = 38$

$x = 2$

$2(2) - 3y = -5$

$4 - 3y = -5$

$-4 \quad\quad -4$

$-3y = -9$

$y = 3$

$(2, 3)$

2. Solve each system of equations using any method.

① $4x + 2y + 3z = 12$

② $2x - 3y + 5z = -7$

③ $6x - y + 4z = -3$

2×③ → $12x - \cancel{2y} + 8z = -6$

① → $4x + \cancel{2y} + 3z = 12$

$16x + 11z = 6$ NEW #1

−3×③ → $-18x + \cancel{3y} - 12z = 9$

② → $2x - \cancel{3y} + 5z = -7$

$-16x - 7z = 2$ NEW #2

NEW #1 $\cancel{16x} + 11z = 6$

NEW #2 $\cancel{-16x} - 7z = 2$

$4z = 8$

$z = 2$ (SUB INTO NEW #1)

$16x + 11(2) = 6$

$16x + 22 = 6$

$16x = -16$

$x = -1$ (SUB z & x INTO ① (or ② or ③))

→ $2y = 10$ (−1, 5, 2)

① $6x + 8y - 6z = 62$

② $10x - 12y - 14z = 14$

③ $12x - 8y + 20z = -68$

NEW #1

①+③ → $18x + 14z = -6$

2×② → $20x - \cancel{24y} - 28z = 28$

−3×③ → $-36x + \cancel{24y} - 60z = 204$

$-16x - 88z = 232$

NEW #2

NEW #1 (8) $18x + 14z = -6$

NEW #2 (9) $-16x - 88z = 232$

$144x + 112z = -48$

$-144x - 792z = 2088$

$-680z = 2040$

$z = -3$ (SUB INTO NEW #1)

$18x + 14(-3) = -6$

$18x = 36$

$x = 2$ (SUB z & x INTO ①)

$6(2) + 8y - 6(-3) = 62$

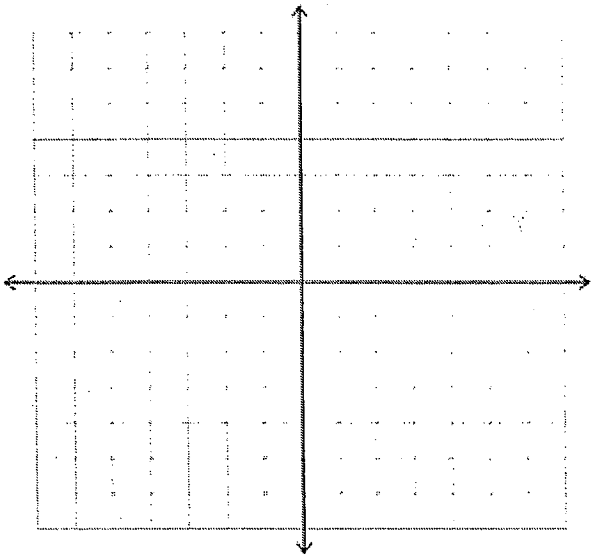
$12 + 8y + 18 = 62$

$8y = 32$

$y = 4$

$(2, 4, -3)$

3. Create a system of linear equations with one solution. Explain why the system has one solution.



One solution - different slopes, lines intersect at a point.

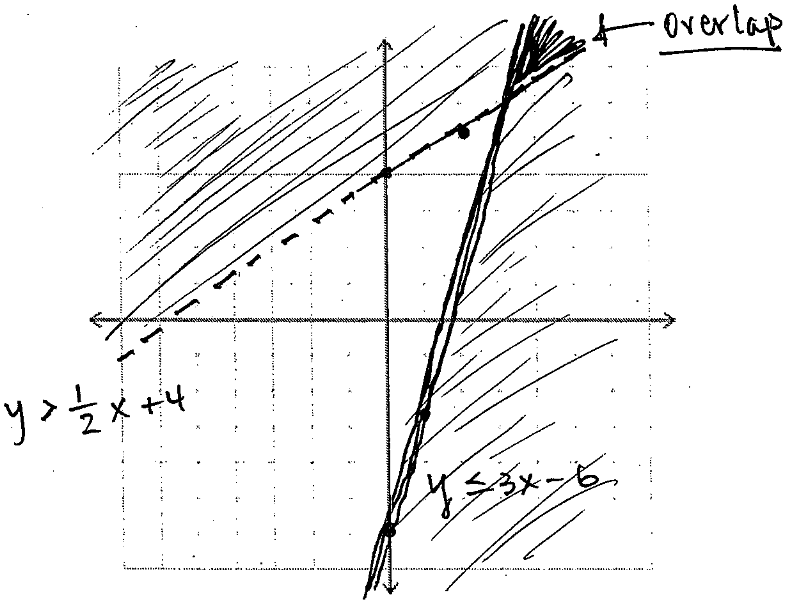
No Solution - same slope, different y-int
lines never intersect
lines are parallel.

Infinitely many solutions -
Same slope, Same y-int.
lines coincide

4. Graph the system of inequalities.

$y > \frac{1}{2}x + 4$

$\frac{1}{3}y \leq x - 2 \rightarrow y \leq 3x - 6$



5. What are three methods for solving a system of linear equations? Which method would you use for the problem below? Give reasons or specific examples to support your answer. Use mathematical reasoning to explain why you did NOT choose the other two methods.

$x = y - 11$
$x - 3y = 1$

I would choose substitution because there is already an isolated variable (x) in the first equation.

Substitution
elimination
graphing

For elimination or graphing, I would need to rearrange terms in one or both equations. Substitution allows me to eliminate the x variable in one step, and solve for y.

6. A drama club earns $1040 from a production. A total of 64 adult tickets and 132 student tickets are sold. An adult ticket costs twice as much as a student ticket. What is the price of each type of ticket? Solve using a system of equations.

$A = 2S$
$64A + 132S = 1040$
$64(2S) + 132S = 1040$ (SUBSTITUTE 2S FOR A)
$128(S) + 132S = 1040$
$260S = 1040$
$S = 4$
$A = 8$

Student tickets cost $4.
Adult tickets cost $8.

7. An amphitheater charges $75 for each seat in section A, $55 for each seat in section B, and $30 for each lawn seat. There are three times as many seats in section B as in section A. The revenue from selling all 23,000 seats is $870,000. How many seats are in each section of the amphitheater?

① $A + B + C = 23{,}000$
② $75A + 55B + 30C = 870{,}000$
③ $B = 3A$

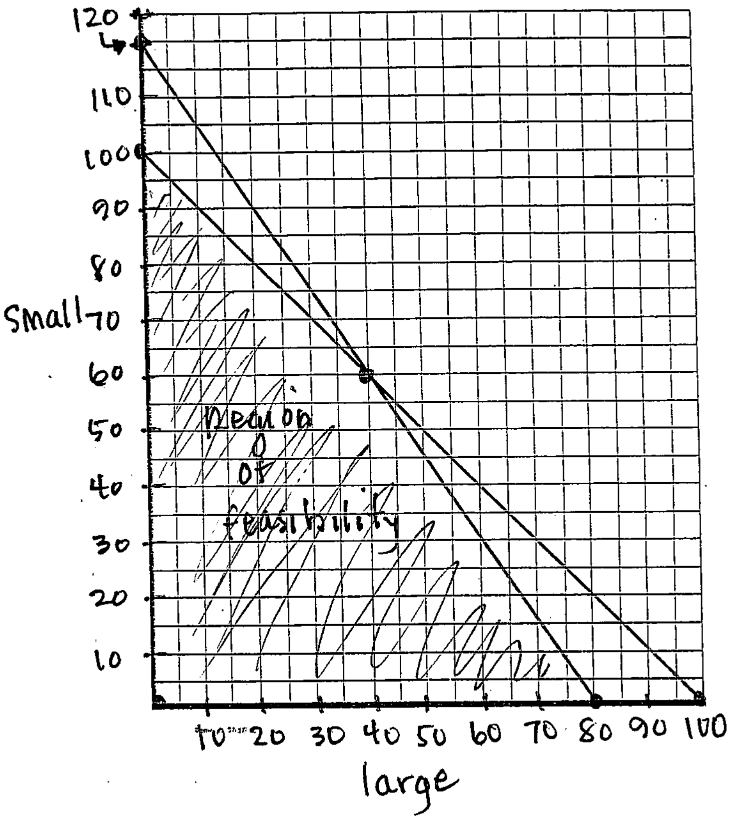
SUBSTITUTE 3A FOR B

$A + 3A + C = 23000$
$75A + 55(3A) + 30C = 870{,}000$

$(-30)\ 4A + C = 23000$
$240A + 30C = 870{,}000$
$\oplus\ -120A - 30C = -690{,}000$
$120A = 180{,}000$
$A = 1500$

OF EACH → $B = 4500$

8. Your club plans to raise money by selling two sizes of fruit baskets. The plan is to buy small baskets for $10 and sell them for $16 and to buy large baskets for $15 and sell them for $25. The club president estimates that you will not sell more than 100 baskets. Your club can afford to spend up to $1200 to buy the baskets. Find the number of small and large baskets you should buy in order to maximize profit.



Follow all of the steps HERE:

OBJECTIVE FUNCTION: MAXIMIZE PROFIT

PROFIT FOR LARGE BASKETS = 25 − 15 = $10

PROFIT FOR SMALL BASKETS = 16 − 10 = $6

$P = 10L + 6S$

CONSTRAINTS

① NUMBER OF BASKETS THAT CAN BE SOLD:

$L + S \leq 100$

INTERCEPTS

(0,100) (100,0)

② BUDGET:

$15L + 10S = 1200$

INTERCEPTS

(0,120) (80,0)

Vertices of region of feasibility: (0,100) (40,60) (80,0) ~~(0,0)~~

X = LARGE

Y = SMALL

PLUG EACH PAIR OF VALUES INTO THE OBJECTIVE FUNCTION.

$P = 10L + 6S$

(0,100) $P = 10(0) + 6(100) = 600$

(40,60) $P = 10(40) + 6(60) = 760$

$\boxed{(80,0)\ P = 10(80) + 6(0) = 800}$

MAX PROFIT OF $800 WHEN 80 LARGE AND ZERO SMALL BASKETS ARE SOLD.