Algebra 2 – Unit 3 Review – Polynomial Functions Name KEY

1. Describe how the degree of a function affects the end behavior of that function.

EVEN DEGREE: GRAPH BEHAVES THE SAME AT BOTH "ENDS"

I.E. AS $X \to -\infty$, $Y \to -\infty$
AS $X \to +\infty$, $Y \to -\infty$

– OR –

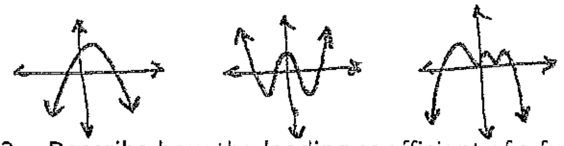
AS $X \to -\infty$ $Y \to +\infty$
AS $X \to +\infty$ $Y \to +\infty$

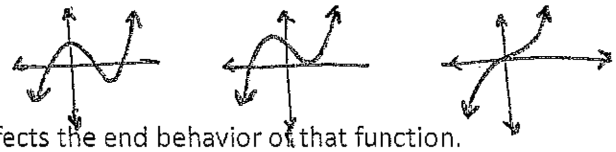
ODD DEGREE: GRAPH AT ONE "END" BEHAVES OPPOSITE TO OTHER "END."

I.E. AS $X \to -\infty$, $Y \to -\infty$
AS $X \to +\infty$, $Y \to +\infty$

– OR –

AS $X \to -\infty$, $Y \to +\infty$
AS $X \to +\infty$, $Y \to -\infty$



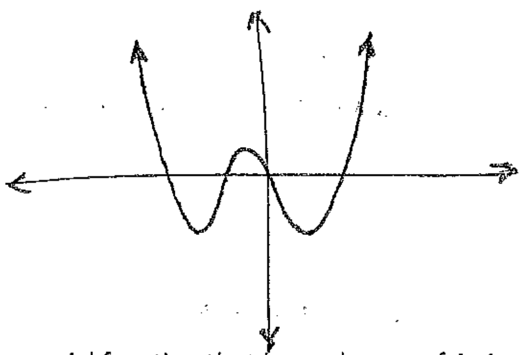


2. Describe how the leading coefficient of a function affects the end behavior of that function.

POSITIVE L.C. (EVEN DEGREE) $Y \to +\infty$ AT BOTH ENDS
(ODD DEGREE) AS $X \to -\infty$, $Y \to -\infty$
AS $X \to +\infty$, $Y \to +\infty$

NEGATIVE L.C. (EVEN DEGREE) $Y \to -\infty$ AT BOTH ENDS
(ODD DEGREE) AS $X \to -\infty$, $Y \to +\infty$
AS $X \to +\infty$, $Y \to -\infty$

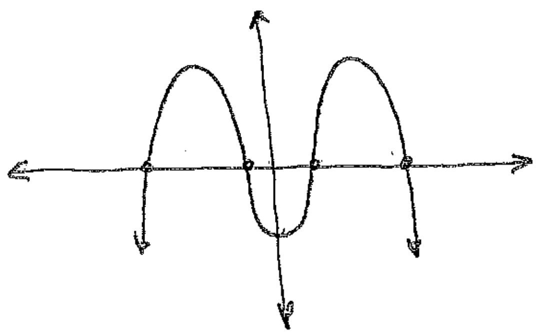
3. Sketch the graph of a polynomial that has an even degree and a positive leading coefficient.



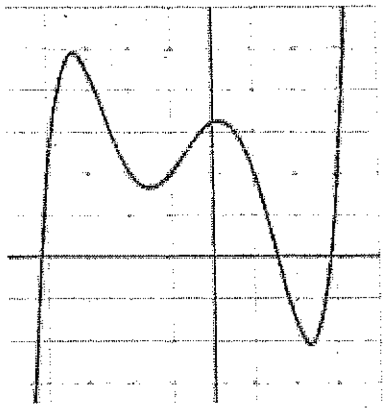
- SAME BEHAVIOR AT BOTH ENDS
- $Y \to +\infty$ AT BOTH ENDS

4. Sketch the graph of a polynomial function that has a degree of 4, 4 real zeros, and a negative leading coefficient.

- THREE (AT MOST) TURNS
- 4 DISTINCT X-INTERCEPTS
- $Y \to -\infty$ AT BOTH ENDS



5. Use the graph below to answer the following questions:



a. Is the degree of this function even or odd? How do you know?

ODD – OPPOSITE BEHAVIOR AT THE ENDS.

b. What type of polynomial is modeled by this graph? How do you know?

QUINTIC (DEGREE 5) – 4 TURNS, DEGREE IS ONE GREATER

c. How many real roots does this polynomial have? How do you know?

(THREE REAL)
FIVE ROOTS – FUNDAMENTAL THEOREM OF ALGEBRA:
POLYNOMIAL OF DEGREE n HAS n SOLUTIONS, THREE X INTERCEPTS

d. How many imaginary roots does this polynomial have? How do you know?

5 – 3 REAL = 2 IMAGINARY

e. Is the leading coefficient of this polynomial positive or negative? How do you know?

POSITIVE – ODD DEGREE, POSITIVE LC, AS $X \to -\infty$, $Y \to -\infty$.

6. Given each polynomial f and a zero of f, use synthetic division, factoring, and/or the quadratic formula to find the other zeros.

a. $f(x) = x^3 + 6x^2 + 5x - 12$; 1

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 5 & -12 \\ & & 1 & 7 & 12 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$X = 1, -3, -4$

$x^2 + 7x + 12 = 0$ FACTORS NICELY!

$(x+3)(x+4) = 0$

$x = -3,\ x = -4$

b. $f(x) = 2x^4 - 9x^3 + 37x - 30$; -2 (DON'T FORGET "0" IN x^2 SPOT!)

$$\begin{array}{r|rrrrr} -2 & 2 & -9 & 0 & 37 & -30 \\ & & -4 & 26 & -52 & 30 \\ \hline 1 & 2 & -13 & \cancel{\text{[illegible]}}\ 26 & -15 & 0 \\ & & 2 & -11 & 15 & \\ \hline & 2 & -11 & 15 & 0 & \end{array}$$

$X = -2, 1, 5/2, 3$

$2x^2 - 11x + 15 = 0$

$(2x - 5)(x - 3) = 0$

$2x - 5 = 0 \qquad x - 3 = 0$

$x = 5/2 \qquad x = 3$

7. Find all real zeros of the function $f(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$.

POSSIBLE RATIONAL ROOTS: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$$\begin{array}{r|rrrrrr} 1 & 1 & -3 & -5 & 15 & 4 & -12 \\ & & 1 & -2 & -7 & 8 & 12 \\ \hline 2 & 1 & -2 & -7 & 8 & 12 & 0 \\ & & 2 & 0 & -14 & -12 & \\ \hline 3 & 1 & 0 & -7 & -6 & 0 & \\ & & 3 & 9 & 6 & & \\ \hline & 1 & 3 & 2 & 0 & & \end{array}$$

$X = 1, 2, 3, -1, -2$

FACTORS NICELY!

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -2,\ x = -1$

8. Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and zeros at -4, 1, and 5.

If a is a root, $x-a$ is a factor.

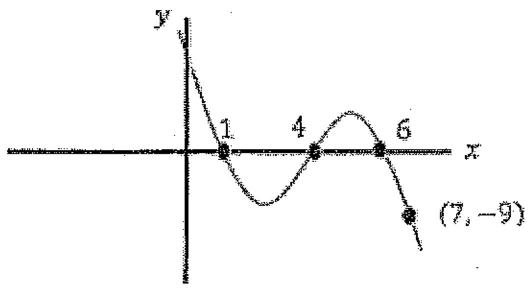
$$f(x) = (x+4)(x-1)(x-5)$$
$$= (x+4)(x^2-6x+5)$$
$$= x^3 - 6x^2 + 5x$$
$$+4x^2 - 24x + 20$$
$$f(x) = x^3 - 2x^2 - 19x + 20$$

9. A quartic (degree 4) polynomial has roots $4+2i$ and $-3-2\sqrt{5}$. What are the remaining roots?

COMPLEX & IRRATIONAL CONJUGATES ...

$4-2i$ and $-3+2\sqrt{5}$

10. Write a polynomial in standard form that could be modeled by the graph below. Please note the function passes through the point (7, -9).



START WITH $f(x) = (x-1)(x-4)(x-6)$

$$= (x-1)(x^2-10x+24)$$
$$= x^3 - 10x^2 + 24x$$
$$- x^2 + 10x - 24$$
$$f(x) = x^3 - 11x^2 + 34x - 24$$

Plug in 7.

$f(7) = 18$, but we want $f(7) = -9$,

So $f(x) = -\frac{1}{2}x^3 + \frac{11}{2}x^2 - 17x + 12$

-OR-

$(x-1)(x-4)(x-6)$

PLUG IN 7:

$(7-1)(7-4)(7-6)$

$6 \cdot 3 \cdot 1 = 18$

$-9 = -\frac{1}{2} \cdot 18$ So you must multiply the entire polynomial by $-\frac{1}{2}$.